REAL BUSINESS CYCLE

1 TABLE OF CONTENTS

2 Real Business Cycle

2.1 Motivation

For descriptive statistics see Slide Handout 1

	Direction	Timing	Volatility
Y	pro cyclical	coincide	= 1
C	pro cyclical	coincide	< 1
G	acyclical		< 1
I	pro cyclical	coincide	>> 1
NX	counter cyclical	leading	
E	pro cyclical	coincide	
U	very counter cyclical	lagging	
W/P	acyclical		< 1
Y/N	pro cyclical	leading	< 1
$\Delta M/M$	pro cyclical	leading	
π	pro cyclical	lagging	
Q	pro cyclical	leading	
i	pro cyclical	lagging	
r	acyclical		<< 1

	St. dev.	Rel. dev.	1st-order autocorr.	Corr. with Y
Y	1.85	1	0.85	1
C	1.17	0.63	0.86	0.80
I	4.41	2.38	0.84	0.62
N	1.95	1.05	0.90	0.82
Y/N	1.12	0.60	0.72	0.66
W/P	0.97	0.47	0.72	–0.06
P	0.94	0.51	0.91	0
$\ln r$	0.70	0.38	0.71	0.02

Source: J. Miao, ch. 14.2. All variables in logs

- Building block of almost all modern DSGE models, being a stochastic neoclassical growth model with variable labour
- Shocks such as technology (Windows Vista good, 7 bad; Finance 2006 good, 2008 bad) and monetary/fiscal (Greenspan Fed changes inflation tolerance)
- Propagation mechanisms: C/I decisions, L decisions, financial mechanisms
    - Exogenous shock is propagated further by endogenous responses

2.2 Intuition of Mechanisms

See also Part IIA Intertemporal Macro section, especially for graphs, Slutsky equation, and precautionary savings

2.2.1 Static Labour/Leisure choice

Set Up and Solve

- Representative household solves $\max_{c,l} \mathcal{L} = \frac{c^{1-\sigma}}{1-\sigma} + \gamma \frac{(1-l)^{\sigma}}{1-\sigma} + \lambda[wl - c]$ s.t. $c = wl$ where $\sigma, \gamma > 0$
    - Normalise price of c to 1
    - Wage exogenous as assume no market power
    - σ is coefficient of relative risk aversion (see Dan notes for derivation).
        - When $\sigma = 1$ this becomes logs per l'Hôpital's rule (with -1) and get CRRA
        - Important for how large smoothing motive is. Larger σ, more risk averse, more precautionary savings, no smoothing (become log)?
- FOCs give
    - $c^{-\sigma} - \lambda = 0$
    - $-\gamma(1-l)^{-\sigma} + w\lambda = 0$
    - $\gamma\left(\frac{c}{1-l}\right)^{\sigma} = w$
- Combine to get $\left(\frac{w}{\gamma}\right)^{\frac{1}{\sigma}}(1-l) = wl$ thus $l = \frac{1}{1+\gamma^{\frac{1}{\sigma}} w^{\frac{\sigma-1}{\sigma}}}$
- Obtain elasticity $\epsilon_{l,w} = \frac{dl}{dw}\frac{w}{l} = \frac{\left(\frac{\sigma-1}{\sigma}\right)\gamma^{\frac{1}{\sigma}} w^{\frac{\sigma-1}{\sigma}}}{1+\gamma^{\frac{1}{\sigma}} w^{\frac{\sigma-1}{\sigma}}} \lesseqgtr 0$ if $1 \lesseqgtr \sigma$

Interpretation

- Do wages increase or decrease labour supply? Decompose into two elements:
    - Substitution Effect: For each hour worked agents will receive more, so l rises (normal)
    - Income Effect: Agent is richer so l falls (ordinary)
- When $\sigma \to 1$ labour supply is independent of w

- Individuals have no wealth with logarithmic utility, thereby income and substitution effect of savings cancel out
- This creates a propagation mechanism
  - Let $Y = Al$ so $w = A$ (i.e. paid MPL)
  - Now change in A has direct effect on Y and indirect effect through l if $\sigma < 1$
  - If substitution effect dominates then $A \rightarrow w \rightarrow l \rightarrow Y$

2.2.2 *Intertemporal Choices*

Set Up and Solve

- Consider representative household maximizing utility over two periods (with discount factor β) subject to intertemporal budget constraint
  - Save/borrow works via K and r ensures S=I i.e. no net saving with representative agent
  - If no K then S(r)=0, if many people then \sum S(r)_{i}=I

$$L = \frac{c_0^{1-\sigma}}{1-\sigma} + \gamma\frac{(1-l_0)^{1-\sigma}}{1-\sigma} + \beta[\frac{c_1^{1-\sigma}}{1-\sigma} + \gamma\frac{(1-l_1)^{1-\sigma}}{1-\sigma}] + \ldots$$
$$+\lambda_0[w_0 l_0 - c_0 - b_0] + \lambda_1[w_1 l_1 + (1+r_1)b_0 - c_1].$$

$$\frac{\partial L}{\partial c_0} = c_0^{-\sigma} - \lambda_0 = 0 \Rightarrow c_0^{-\sigma} = \lambda_0 \quad (1)$$
$$\frac{\partial L}{\partial c_1} = c_1^{-\sigma} - \lambda_1 = 0 \Rightarrow \beta c_1^{-\sigma} = \lambda_1 \quad (2)$$
$$\frac{\partial L}{\partial l_0} = -\gamma(1-l_0)^{-\sigma} - w_0\lambda_0 = 0 \Rightarrow \gamma(1-l_0)^{-\sigma} = w_0\lambda_0 \quad (3)$$
$$\frac{\partial L}{\partial l_1} = -\beta\gamma(1-l_1)^{-\sigma} - w_1\lambda_1 = 0 \Rightarrow \beta\gamma(1-l_1)^{-\sigma} = w_1\lambda_1 \quad (4)$$
$$\frac{\partial L}{\partial b_0} = -\lambda_0 + \lambda_1(1+r_1) \Rightarrow \lambda_0 = \lambda_1(1+r_1). \quad (5)$$

+ the two constraints!

Interpretation

- (1) (2) (5) give <u>Euler equation</u>: $c_0^{-\sigma} = (1+r_1)\beta c_1^{-\sigma}$ thus $\frac{c_1}{c_0} = [\beta(1+r_1)]^{1/\sigma}$
  - Changes in future interest rate will change decision to save/invest. This can be due to shock today that persists tomorrow or unanticipated future shock (i.e. expected A)
    - Substitution Effect: when r increases 'price' of c_1 (or opportunity cost of c_2) falls so agents prefer to save more
    - Income Effect: If agents are net savers/borrowers then income rises/falls so want to smooth consumption by consuming more/less today
- (3) (4) (5) give <u>Euler equation</u>: $\frac{1-l_1}{1-l_0} = \left[\beta(1+r_1)\frac{w_0}{w_1}\right]^{1/\sigma}$
  - Shock may be amplified by household decision to work and invest
    - Rise in $\frac{w_0}{w_1}$ means more work today relative to tomorrow
    - Rise in r_1 means more work today relative to tomorrow

2.2.3 *Uncertainty*

Set Up and Solve

- Assume: Individual lives for two periods (0,1) where they consume/save income Y_0, Y_1. Income Y_1 is stochastic where $Y_1 = \begin{cases} Y_1^H & \text{prob } \pi_1 \\ Y_1^L & \text{prob } 1-\pi_1 \end{cases}$ where $Y_1^H > Y_1^L$
- Thus solve $\max_{C_0,B_0,C_1} \{u(C_0) + \beta[\pi_1 u(C_1^H) + (1-\pi_1)u(C_1^L)]\} = \{u(C_0) + \beta E_0[u(C_1)]\}$

- s.t. $C_0 + B_0 = Y_0$; $C_1^H = Y_1^H + B_0(1+r)$; $C_1^L = Y_1^L + B_0(1+r)$

- Solving Lagrange $u'(C_0) = \beta[\pi_1 u'(C_1^H) + (1-\pi_1)u'(C_1^L)](1+r) = \beta(1+r)E_0[u'(C_1)]$
- Reduction in utility in period 0 is exactly offset by increase in expected discounted utility in period 1

2.3 RBC Model - See Also Supo 1Feedback!!!

2.3.1 *General*

Set Up

- Firms: price taking and can model with single representing (large number, identical)
- Households: infinitely-lived/PIH and can model with single representing (large number, identical)
- Market: Walrasian (i.e. perfect competition, flexible prices) and classical dichotomy (use real prices)
- Inputs in production are K, L, Z and income can be used for C, I
- Fraction δ of capital depreciates each period

Firm

- Maximize expected lifetime discounted profits but since there is no intertemporal trade-off this is equivalent to simply maximizing period-by-period profits
- $\pi_t = \max_{K_t, L_t}[Z_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t^K K_t]$ where $\alpha \in (0,1)$
  - Where Z_t corresponds to shocks
  - Assume K_t is owned and rented out by households
- FOCs give $w_t = (1-\alpha)Z_t K_t^\alpha L_t^{-\alpha}$ and $r_t^K = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha}$ both depend on Z_t

Household

- Maximize expected lifetime utility: $U = E_0[\sum_{t=0}^{\infty} \beta^t u(c_t, 1-l_t)]$ where $\beta \in (0,1)$
  - $E_0\{g(Z_t)\}$ means expectations of $g(Z_t)$ conditional on information at 0
  - Time endowment normalised to 1
- Subject to each time period's budget constraint: $c_t + k_{t+1} = w_t l_t + (1 + r_t^K - \delta)k_t$
  - $c_t + x_t = w_t l_t + r_t^K k_t$ is standard, recalling households own capital
    - But… own different firm than they work for, else problem becomes different
  - $k_{t+1} = (1-\delta)k_t + x_t$ is Solow capital law of motion
- Thus $L = E_0[\sum_{t=0}^{\infty} \beta^t (u(c_t, 1-l_t) + \lambda_t(w_t l_t + (1 + r_t^K - \delta)k_t - c_t - k_{t+1}))]$ with FOCs:
  - $\frac{dL}{dc_t} = E_0[\beta^t u_c(c_t, 1-l_t) - \beta^t \lambda_t] = 0$ thus $E_0[u_c(c_t, 1-l_t)] = E_0[\lambda_t]$
  - $\frac{dL}{dl_t} = E_0[-\beta^t u_l(c_t, 1-l_t) + \beta^t \lambda_t w_t] = 0$ thus $E_0[u_l(c_t, 1-l_t)] = E_0[\lambda_t w_t]$
  - $\frac{dL}{dk_{t+1}} = E_0[-\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}(1 + r_{t+1}^K - \delta)] = 0$ thus $E_0[\lambda_t] = \beta E_0[\lambda_{t+1}(1 + r_{t+1}^K - \delta)]$
    - Lagrange multiplier is itself uncertain!

Expectations

- At period t, variables dates t are known but future variables taken with expectations
  - $E_0[x_t] = E_t[x_t] = x_t$ and $E_0[x_{t+1}] = E_t[x_{t+1}]$
- Maximizing lifetime utility at 0 is equivalent to maximizing period-by-period with RatEx
  - Know will be rational in any period t and rational is right on average, thus can ignore shocks
- FOCs become [1] $u_c(c_t, 1-l_t) = \lambda_t$; [2] $u_l(c_t, 1-l_t) = w_t \lambda_t$; [3] $\lambda_t = \beta E_t[\lambda_{t+1}(1 + r_{t+1}^K - \delta)]$

- Combining [1] and [3] get $u_c(c_t, 1-l_t) = \beta E_t[\lambda_{t+1}(1 + r_{t+1}^K - \delta)]$

- Euler equation (intertemporal condition): If household saves unit of extra consumption in t at expected interest rate r_t+1-\delta, reduction in utility in t is exactly offset by increase in expected discounted utility in t+1

- Combining [1] and [2] get $w_t u_c(c_t, 1-l_t) = u_l(c_t, 1-l_t)$ thus $\frac{u_l(c_t,1-l_t)}{u_c(c_t,1-l_t)} = w_t$
  - Euler equation (intra-temporal condition): If the household works more than LHS represents the disutility from working while the right-hand side corresponds to the marginal benefit of working more

- Household

$$c_t^{-\sigma} = \beta E_t\left[c_{t+1}^{-\sigma}\left(1+r_{t+1}^K-\delta\right)\right] \quad (1)$$

$$\frac{\gamma(1-l_t)^{-\sigma}}{c_t^{-\sigma}} = w_t \quad (2)$$

$$c_t + k_{t+1} = \left(1+r_t^K-\delta\right)k_t + w_t l_t \quad (3)$$

- Firm

$$r_t^K = \alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha} \quad (4)$$

$$w_t = (1-\alpha) Z_t K_t^{\alpha} L_t^{-\alpha} \quad (5)$$

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \quad (6)$$

- Markets

$$Y_t = C_t + I_t \text{ (goods)} \quad (7)$$

$$X_t = K_{t+1} - (1-\delta)K_t = S_t \text{ (credit)} \quad (8)$$

$$L_t = l_t \text{ (labour)} \quad (9)$$

  - 9 equations, 8 unknowns thus can even remove one
    - Irl solve r then K and C last – see supo 1 feedback
    - Note L^S and L^d are not interchangeable but need to work out equilibrium price w

- Note have output equation $c_t + S_t = r_t^K K_t + w_t L_t$ thus $c_t + X_t = MPK_t * K_t + MPL_t * L_t = Y_t$
  - Household budget constraint equivalent to national income identity
- See how shocks propagate:
  - Z_t improves MPL, raising wages, changing labour supply (depending on substitution effect)
  - Z_t improves MPK, which might increase investment
- In general system is highly non-linear and hard to solve
  - Thus consider special case where $\sigma = 1$ (log utility), $\gamma = 1$ (do not value leisure so $l_t = 1$), and $\delta = 1$ (full depreciation of capital)

2.3.2 *Special Case*

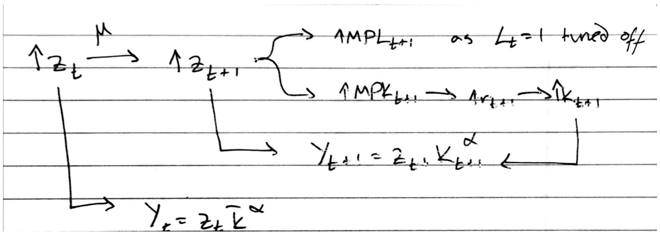
Assumptions

- Full depreciation of capital $\delta = 1$
- One period utility function $u_t = \ln c_t$ [i.e. $\sigma \to 1$ and $\gamma = 0$ so labour supply is inelastic]
- Thus equilibrium conditions become
  - Resource constraint $C_t + X_t = Y_t$ thus $C_t + K_{t+1} = Z_t K_t^{\alpha}$
  - Euler equation $\frac{1}{C_t} = \beta E_t\left[r_{t+1}^K \frac{1}{C_{t+1}}\right]$ thus $1 = \beta E_t\left[\frac{C_t}{C_{t+1}} \alpha Z_{t+1} K_{t+1}^{\alpha-1}\right]$

Solving

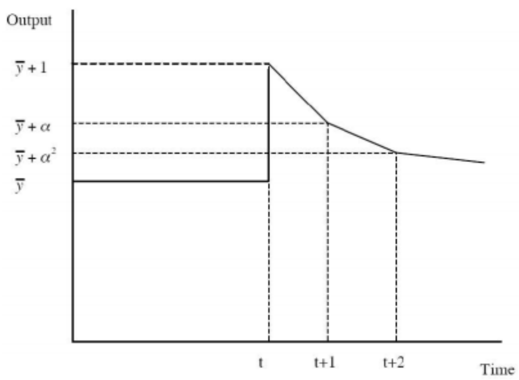
- Guess and Verify that $C_t = \theta Z_t K_t^{\alpha}$ where $\theta \in (0,1)$
  - Consumption is constant fraction of output thus constant saving rate thus back to Solow!
- Resource constraint now $1 = \beta E_t\left[\frac{\theta Z_t K_t^{\alpha}}{\theta Z_{t+1} K_{t+1}^{\alpha}} \alpha Z_{t+1} K_{t+1}^{\alpha-1}\right] = \alpha\beta E_t\left[\frac{Z_t K_t^{\alpha}}{(1-\theta) Z_t K_t^{\alpha}}\right] = \frac{\alpha\beta}{1-\theta}$ thus $\theta = 1 - \alpha\beta$

- Substituting back in for results:
  - $C_t = (1-\alpha\beta)Z_t K_t^{\alpha}$ thus $c_t = \ln(1-\alpha\beta) + z_t + \alpha k_t$ [verifies initial guess!]
  - $K_{t+1} = \alpha\beta Z_t K_t^{\alpha}$ thus $k_t = \ln(\alpha\beta) + z_t + \alpha k_t$
  - $Y_t = Z_t K_t^{\alpha}$ thus $y_t = z_t + a k_t$
  - $r_t = \alpha Z_t K_t^{\alpha-1}$ and $w_t = (1-\alpha) Z_t K_t^{\alpha}$
    - $S_t = X_t = K_{t+1}$ and $L_t = 1$
- Now model technology shock as AR(1) process: $\ln(Z_t) = \mu \ln(Z_{t-1}) + \epsilon_t$ thus $z_t = \mu z_{t-1} + \epsilon_t$
  - Thus Y_t becomes MA(1) process. See Vasco PhD for ARMA(1) critique
- Note two sources of persistence:
  - Exogenous technology persistence: If more output than expected today, tomorrow will also be relative high
  - Endogenous consumption smoothing: If more output than expected today, save for future consumption. This builds up capital stock, increasing future output



Dynamics

- Thus $Y_{t+1} = Z_{t+1}K_{t+1}^{\alpha} = Z_{t+1}(\alpha\beta Y_t)^{\alpha}$ and $y_{t+1} = z_{t+1} + \alpha\ln(\alpha\beta) + \alpha y_t$
  - Z_{t+1} is stochastic shock and $a y_t$ persistent dynamics
- Steady state: $\bar{y} = \frac{\alpha\ln(\alpha\beta)}{1-\alpha}$ thus $y_{t+1} - \bar{y} = \alpha(y_t - \bar{y}) + z_{t+1}$
  - Where the economy would converge if production fixed at mean
  - Normalize $z = 1$ or $\bar{z} = 0$
- Let $\mu = 0$ so have white noise shock now…
  - $z_t = 0$; $z_{t+1} = 1$; $z_{t+2} = 0$; $z_{t+3} = 0$
  - $y_t = \bar{y}$; $y_{t+1} = \bar{y} + 1$; $y_{t+1} = \bar{y} + \alpha$ $y_{t+3} = \bar{y} + \alpha^2$
  - This creates cycle (isolates endogenous persistence, as save more for future periods, save by increasing K, motive gets smaller over time)



- Z_t is main driver of model, but problem is that productivity is just growth residual so unexplained
  - First differences $\Delta y_t = \Delta z_t + \alpha\Delta k_t + (1-\alpha)\Delta l_t$ thus $\Delta z_t = \Delta y_t - (\alpha\Delta k_t + (1-\alpha)\Delta l_t)$
  - TFP correlation with output growth 0.85 in US
  - 2008 interpretation as fall in Z_t is stupid since don't just forget how to do things with same

Applying IRL

- For parametrization see slides
- Qualitative predictions are good: y_t, c_t, k_{t+1} all comove with productivy and thus positively with each other
- But quantitative are wrong, though these can be somewhat fixed in more general models
  - I is as cyclical as C in model but more irl
    - Because assumed full depreciation
  - Total labour fixed over business cycle
    - Because assumed no leisure-labour choice
  - IRL observe output shock is hump-shaped with peak occurring after 1-2 quarter and half-life of shock is 2.5 years
    - RBC only does these if tech is very persistent (high μ) does not generate enough endogenous persistence
- Objections: Ignores demand shock; needs very persistent exogenous shocks since weak propagation mechanism; highly variable employment only if assume unreasonably high elasticity of labour
  - Also note there is no role for stabilisation policy atm!

3 Demand Driven Fluctuations (Mankiw & Weinzerl, 2011)

3.1 Set Up

Households

- Large number of identical households: $U = u(C_1) + \beta u(C_2)$
- Budget constraint states present value of lifetime nominal consumption cannot exceed that of income of profits accruing from ownership of firms: $P_1(\Pi_1 - C_1) + \frac{P_2(\Pi_2 - C_2)}{1+i_1} = 0$
  - Implicit here is the assumption of a bond market that allows to borrow/save

Firms

- Large number of identical firms: $\max_{K_2} P_1\Pi_1 + \frac{P_2\Pi_2}{1+i_1}$
  - where $\Pi_t = Y_t - I_t$; $Y_t = A_t K_t$; K_1 is given
- Capital is only factor of production and chosen through investment decisions
  - Firms now own the capital stock
- There are only two periods so $I_2 = 0$ and assume full depreciation so $K_2 = I_1$

Money Market and Monetary Policy

- <u>Cash-In-Advance Constraint</u>: Households hold money to purchase goods. Thus money market equilibrium condition $M_t = P_t C_t$
- Central Bank has two policy variables: i_1 (or M_1 directly) and M_2 (as no i_2 exists)

Aggregate Demand and Supply

- Aggregate Demand: $C_t + I_t$
- Aggregate Supply constrained by potential output $Y_t \leq A_t K_t$
  - Under full employment this holds with equality. When AD is insufficient to employ all productive resources, equ. output is smaller than potential!

3.2 SOLVING

- *Households* max utility subject to budget constraint. FOC gives Euler equation: $\frac{u'(C_1)}{\beta u'(C_2)} = (1+i_1)\frac{P_1}{P_2}$
- *Firms* choose investment to max profits giving FOC: $A_2 = \frac{P_1}{P_2}(1+i_1)$
- *Output* market clears at every period: $C_1 + K_2 = Y_1$ and $C_2 = Y_2$
- *Money* market clears at every period $P_t C_t = M_t$

3.2.1 *ASSUMING FLEXIBLE PRICES*

- Prices adjust to guarantee $Y_t = A_t K_t$
  - Thus $C_1 + K_2 = A_1 K_1$ so $C_1 = A_1 K_1 - K_2$ (sub into market clearing condition)
  - Thus $A_2 = \frac{u'(A_1 K_1 - K_2)}{\beta u'(A_2 K_2)}$ (sub into Euler)
  - No have implicitly defined K_2 in terms of K_1 and parameters. Now solve for C_1, C_2, Y_2
- Assuming CRRA utility with elasticity of substitution σ [$u(C_t) = \frac{C_t^{1-1/\sigma}-1}{1-1/\sigma}$] we get…

$$
\begin{aligned}
C_1 &= \frac{\left(\frac{1}{\beta A_2}\right)^\sigma A_2}{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}(A_1K_1) \\
C_2 &= \frac{A_2}{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}(A_1K_1) \\
I_1 &= K_2 = \frac{1}{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}(A_1K_1) \\
Y_1 &= A_1K_1 \\
Y_2 &= \frac{A_2}{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}(A_1K_1)
\end{aligned}
\qquad
\begin{aligned}
P_1 &= \frac{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}{(A_1K_1)}\frac{M_2}{1+i_1} \\
P_2 &= \frac{1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2}{A_2(A_1K_1)}M_2 \\
M_1 &= \left(\frac{1}{\beta A_2}\right)^\sigma A_2\frac{M_2}{1+i_1}
\end{aligned}
$$

- Money Neutrality: Real equilibrium quantities do not depend on MP instruments / price

Comparative Static

- Consider productivity shock tomorrow that induces decline in household spending today (i.e. A_2). That is adverse "animal spirits" as purely demand disturbance means $A_1 K_1$ remains same!
- Iff $\sigma < 1$ income effect dominates so lower A_2 causes households to feel poorer and C_1 falls
  - $C_1 + I_1 = A_1 K_1$: C_1 falls and but offset by I_1 rising

3.2.2 *ASSUMING STICKY PRICES*

- That is short-run sticky prices: P_1 fixed but P_2 flexible
- Nominal rigidity may prevent full employment of capital: $C_1 + I_1 < A_1 K_1$

$$
\begin{aligned}
C_1 &= \left(\frac{1}{\beta A_2}\right)^\sigma A_2\frac{M_2}{(1+i_1)P_1} \\
C_2 &= A_2\frac{M_2}{(1+i_1)P_1} \\
I_1 &= K_2 = \frac{M_2}{(1+i_1)P_1} \\
Y_1 &= \left(1+\left(\frac{1}{\beta A_2}\right)^\sigma A_2\right)\frac{M_2}{(1+i_1)P_1} \\
Y_2 &= A_2\frac{M_2}{(1+i_1)P_1} \\
P_2 &= \frac{(1+i_1)}{A_2}P_1
\end{aligned}
$$

- <u>Money Non-Neutrality</u>: Y_1 is determined by Aggregate Demand curve as have negative relationship between Y_1 and P_1 [depends on $\frac{M_2}{(1+i_1)P_1}$]

Comparative Static

- Consider again demand shock A_2:
  - Consumption falls in both periods and investment stays constant!
  - Thus Y_2 falls and Y_1 falls too (even though potential output is unchanged)
  - I.e. Weak aggregate demand causes resources K_1 in first period to become idle
- Now have a role for policy! MP can restore full employment by responding to present and future technology
  - $Y_1 = \left(1 + \left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right) \frac{M_2}{(1+i_1)P_1}$
  - So For $Y_1 = A_1 K_1$ we need $\frac{M_2}{(1+i_1)P_1} = \frac{1}{\left(1+\left(\frac{1}{\beta A_2}\right)^{\sigma} A_2\right)} A_1 K_1$
  - A decline in A_2 leads to MP easing. If large enough we have $i_1 < 0$ so ZLB

4 New Keynesian Model

- Problem was not empirical evidence against Keynesian Theories (supply shocks is permissible in model) but weakness in theories themselves (not enough detail of supply slide to be useful; Lucas critique of lack of micro foundation)
- Focuses on imperfect competition in the market for goods and cost of changing prices (menu costs), resulting in flexible-price equilibrium output being socially suboptimal and money non-neutrality

4.1 Basic Model

Key Assumptions

- Continuum of households, each owning a firm they do not work for (easier if households don't internalise firm decisions)
- Firm produces differentiated good that are imperfect substitute (elasticity of substitution $\eta > 1$; else firm would charge infinite prices), creating monopolistic competition
- Labour market is perfectly competitive and elasticity of supply $\gamma > 1$

Households

- Solve to maximize $\max_{C_i, L_i} \left(C_i - \frac{1}{\gamma} L_i^{\gamma}\right)$ $s.t.\ PC_i = P\Pi_i + WL_i$ that is $\max \mathrm{U} = \Pi_i + \frac{W}{P} L_i - \frac{1}{\gamma} L_i^{\gamma}$
- Get FOC: $L_i = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$ thus $\ln L_i = l_i = \frac{1}{\gamma-1} \ln\left(\frac{W}{P}\right)$
  - Note as $\gamma \to 1$ get infinitely elastic labour supply

Firms

- Demand for good of firm i: $Q_i = Y\left(\frac{P_i}{P}\right)^{-\eta}$ thus $q_i = y - \eta(p_i - p)$ or $p_i = p + \frac{y}{\eta} - \frac{1}{\eta} q_i$
  - Degree of substitution determines market power. Higher η means consumers are more willing to give up good in response to price rise
    - $\eta \to \infty$ get perfect competition
    - $\eta \to 1$ is pure monopoly

- Production function simply $Q_i = L_i$
- Solve to maximize real profits $\max_{\frac{P_i}{P}} Q_i \left(\frac{P_i}{P} - \frac{W}{P}\right) = \max_{\frac{P_i}{P}} Y \left(\frac{P_i}{P}\right)^{-\eta} \left(\frac{P_i}{P} - \frac{W}{P}\right)$
    - Firms do not account how they affect aggregate P
- Get FOC wrt $\frac{P_i}{P}$ that is $Y \left(\frac{P_i}{P}\right)^{-\eta} - \eta Y \left(\frac{P_i}{P} - \frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\eta-1} = 0$ thus $\frac{P_i}{P} = \left(\frac{\eta}{\eta-1}\right)\frac{W}{P}$
    - Where $\frac{\eta}{\eta-1}$ is mark up
- Scale single firm to aggregate via $Y = \left[\int Y(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$ as $\epsilon \to \infty$. This makes it micro-founded equation, where Y is average or aggregate (same because of normalisation)

Symmetric Equilibrium

- We have two markets requiring equilibriums $L^S = L^D$ and $AD = AS$
    - Goods has [AS] $Y = \int Q_i = \int L_i = L^D$ and [AD] $Y = \frac{M}{P}$ (due to money neutrality)
        - Assume firms produce same quantity $Q_i = L = Y$
        - Assume firms charge same price $P_i = P$
    - Labour has $L^S = \frac{W}{P}$
        - Assume households supply same labour $L_i = L$
- Hence solve:
    - For output $\frac{P_i}{P} = \frac{\eta}{\eta-1}\frac{W}{P} = \frac{\eta}{\eta-1}Y^{\gamma-1}$ thus $Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}} < 1$ that is less than potential!
    - Likewise, price level: $P = \frac{M}{Y} = \frac{M}{\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}}$

4.1.1 *IMPLICATIONS*

- Socially optimal equilibrium output is $Y^* = 1$ (corresponding to $\eta \to \infty$) but under monopolistic competition $Y^N < Y^*$ and real wage is paid less than MPL
    - Thus have asymmetric welfare effects: If $Y^N < Y^*$ then shock moving $Y > Y^N$ can be good! By contrast under perfect competition shocks are always bad.
        - This hinges on $Y^N \neq Y^*$. Most models explicitly don't assume this
    - Intuitively, the fact that producers face downward-sloping demand curves means that the marginal revenue product of labor is less than its marginal product. As a result, the real wage is less than the marginal product of labor: from (6.55) (and the fact that each Pi equals P in equilibrium), the real wage is (η − 1)/η; the marginal product of labor, in contrast, is 1. This reduces the quantity of labor supplied, and thus causes equilibrium output to be less than optimal.
    - gap between the equilibrium and optimal levels of output is greater when producers have more market power (that is, when ηi s lower) and when labor supply is more responsive to the real wage (that is, when γ is lower).
  - MANKIW 1985: Asymmetric welfare effects
  - demand curve for each good, Y(Pi/P)−η, shifts out. Since firms are selling at prices that exceed marginal costs, this change raises profits, and so increases households' welfare. Thus under imperfect competition, pricing decisions have externalities, and those externalities operate through the overall demand for goods. This externality is often referred to as an aggregate demand externality (Blanchard and Kiyotaki, 1987)
- Real rigidity: responsiveness of relative price to changes in output captured by γ

- $\frac{P_i}{P} = \frac{\eta}{\eta-1} Y^{\gamma-1}$ thus $p_i - p = k + (\gamma - 1)y$
- Show money neutrality by noting $\frac{dW}{dM} = 1$ and $\frac{dP}{dM} = \frac{1}{\left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}}$

p∗i− p takes the form c + φy. That is, it states that a price-setter's optimal relative price is increasing in aggregate output. In the particular model we are considering, this arises from increases in the prevailing real wage when output rises. But in a more general setting, it can also arise from increases in the costs of other inputs, from diminishing returns, or from costs of adjusting output.

4.1.2 Menu Costs and Money Non-Neutrality

- If prices are flexible money is neutral. Imperfect competition alone is not enough
    - $\Delta M > 0 \rightarrow$ all change $p(i)$ [preferred to changing q] $\rightarrow \frac{p(i)}{p}$ no change $\rightarrow \Delta Y = \Delta\left(\frac{W}{P}\right) = 0$
- Also need menu costs to have money non neutrality
    - δ can adjust p (and q but just p is optimal)
    - $(1-\delta)$ cannot adjust p but can q (sub-optimally)
    - Rise in δ, more firms change p not q, less change in y
    - $\Delta Y = \frac{\Delta M}{P}$ and money is non-neutral
- There are many theories of menus costs. We will focus on Calvo theory (randomly assigns who falls in δ) or that there exists cost $Z \geq 0$ that has effect if large relative to ΔM

- [1] Imperfect Competition and [2] nominal rigidities allow NK (monetary affects output) and thus are complements
    - If not [2] get money neutrality. Money shock -> firms want to adjust via P not Q -> firms adjust Q -> Change in y
    - If not [1] firms not price makers but takers. Hence cannot have theory of price setting as output always 1 regardless of price
    - If not [1] or [2] have have symmetric equilibrium as all firms face same optimum
- But… to satisfy data irl also need [3] real rigidities as amplification mechanism because nominal rigidities are too small (Ball & Romer)
    - Do so through labour as nominal wage may be stuck, thus unemployment more volatile. That is output doesn't respond fully because labour supply dampens response

When do firms not want to change prices? Symmetric Nash Equilibrium

- Assume others expect no one else will change price
- Thus firms simply choose not to change their price if $\Pi_{ADJ} - \Pi_{FIX} = \delta < Z$
    - If relative profit from changing price when no one else does (δ) is smaller than menus cost (Z) then firm i does not change price
- Recall $\Pi_i = Q_i\left(\frac{P_i}{P} - \frac{W}{P}\right) = Y\left(\frac{P_i}{P}\right)^{-\eta}\left[\frac{P_i}{P} - Y^{\gamma-1}\right] = \frac{M}{P}\left(\frac{P_i}{P}\right)^{1-\eta} - \left(\frac{M}{P}\right)^{\gamma}\left(\frac{P_i}{P}\right)^{-\eta}$
    - Substituting in Labour Market equilibrium condition $\frac{W}{P} = L^{\gamma-1} = Y^{\gamma-1} = \left(\frac{M}{P}\right)^{\gamma-1}$
- If firm does not change price $P_i = P$
    - Thus $\Pi_{FIX} = \frac{M}{P} - \left(\frac{M}{P}\right)^{\gamma}$

- If firm changes its price it sets $\frac{P_i}{P} = \left(\frac{\eta}{\eta-1}\right)\frac{W}{P} = \left(\frac{\eta}{\eta-1}\right)\left(\frac{M}{P}\right)^{\gamma-1}$
    - Thus $\Pi_{ADJ} = \frac{M}{P}\left(\frac{P_i}{P}\right)^{1-\eta} - \left(\frac{M}{P}\right)^{\gamma}\left(\frac{P_i}{P}\right)^{-\eta} = \frac{1}{\eta-1}\left(\frac{\eta}{\eta-1}\right)^{-\eta}\left(\frac{M}{P}\right)^{\eta+\gamma-\eta\gamma}$
- Empirical Evidence on Nominal Rigidities
- Money shocks have real effects on output (Romer & Romer, 1989)
- Prices adjust infrequently (Carlton, 1986)
- Question remain as to whether menu costs are large enough to explain real effect of large money shocks (Ball & Romer, 1990). More realistic is that large δ so money almost neutral

4.2 New Keynesian Phillips Curve

- See Part IIA essay plan and revision notes

4.2.1 *Motivation*

- Old expectations Augmented Phillips Curve: $\pi_t = E_{t-1}\pi_t + \alpha(y_t - \bar{y})$
    - Where $\bar{y}$ is natural rate of output
    - Implies only unanticipated inflation results in output fluctuations
- Then RatEx revolution led by Lucas, Sargent, Wallace and co. proposed that individuals do not make systematic forecast mistakes: $E(\epsilon) = E(E_{t-1}X - X) = 0$ thus $E(X) = \bar{X}$
    - See Policy Ineffectiveness notes in Part IIA
    - But… conflict with empirical VAR evidence
- Reaction has been New Keynesian model in which prices and wages do not instantaneously adjust

Old/Inertial Phillips Curve

- Note three equations:
    - [1] Optimal labour supply of representative household is $\ln(L_i) = \frac{1}{\gamma-1}\ln\left(\frac{W}{P}\right)$
    - [2] Optimal pricing equation of firms is $\ln\left(\frac{P_i}{P}\right) = \ln\left(\frac{\eta}{\eta-1}\right) + \ln\left(\frac{W}{P}\right)$
    - [3] Production function is $\ln(Q_i) = \ln(L_i)$
- Now use these to derive:
    - Combining [2] and [1] get [A]: $\ln\left(\frac{P_i}{P}\right) = \ln\left(\frac{\eta}{\eta-1}\right) + (\gamma-1)\ln(L_i)$
    - In equilibrium $P_i = P$ thus [B]: $0 = \ln\left(\frac{\eta}{\eta-1}\right) + (\gamma-1)\ln(\bar{L})$
    - Subtracting [A]-[B] get $\ln\left(\frac{P_i}{P}\right) = (\gamma-1)[\ln(L_i) - \ln(\bar{L})]$
    - Rearrange $\ln(P_i) = \ln(P) + (\gamma-1)[\ln(L_i) - \ln(\bar{L})]$
    - Taking firm expectations of $\ln(P)$ and aggregating: $\ln(P) = E\ln(P) + (\gamma-1)[\ln(Y) - \ln(\bar{Y})]$
    - Subtracting $\ln(P_{t-1})$ in both sides thus get $\pi_t = E_{t-1}\pi_t + (\gamma-1)(y_t - \bar{y})$
- This model is outdated because
    - lacks micro-foundations of forward-looking behaviour of price setters
    - No role for monetary policy to aid speed at which economy returns to equilibrium

4.2.2 *Calvo Fairy*

Set Up

- Each period a fraction δ of firms is able to set own price, which it does taking into account expected average market price and future expected demand
    - $\ln(P_t) = \delta\ln(P_t^*) + (1-\delta)\ln(P_{t-1})$
    - Rewrite as $\pi_t = \delta\pi_t^*$ where $\pi_t^* = \pi_t + (\gamma-1)x_t$ and $x_t = y_t - \bar{y}$

- Firms discount future are rate ϕ

Solving

- Key insight: Decision today will also affect future so long until we can choose price again ($t+1$ with p=$(1-\delta)$; $t+2$ with p=$(1-\delta)^2$). Thus π_t^* isn't just corrected for t but also future weighted by p
- Solve as follows (See slides handout 7 for complex derivation)
    - Note {1} $\pi_t^* = E_t \sum_{i=0}^{\infty}(1-\delta)^i\phi^i\pi_{t+i}^* = E_t \sum_{i=0}^{\infty}(1-\delta)^i\phi^i[\pi_{t+i} + (\gamma-1)x_{t+i}] = \pi_t + (\gamma-1)x_t + (1-\delta)\phi[E_t\pi_{t+1} + (\gamma-1)E_t x_{t+1}] + (1-\delta)^2\phi^2[E_t\pi_{t+2} + (\gamma-1)E_t x_{t+2}] \dots$
    - Using $\pi_t = \delta\pi_t^*$ also note {2} $\pi_t = \delta\{\pi_t + (\gamma-1)x_t + (1-\delta)\phi[E_t\pi_{t+1} + (\gamma-1)E_t x_{t+1}] + (1-\delta)^2\phi^2[E_t\pi_{t+2} + (\gamma-1)E_t x_{t+2}] \dots\}$
    - See slides for complex derivation: Multiply by $(1-\delta)\phi$; Evaluate in $t+1$; Take expectations; Subtracting {1}-{2} and delete common terms
    - Obtain $(1-\delta)\pi_t - (1-\delta)\phi E_t\pi_{t+1} = \delta(\gamma-1)x_t$
    - Rearrange to $\pi_t = \phi E_t\pi_{t+1} + \frac{(\gamma-1)\delta}{1-\delta}x_t$
        - $\delta = 0.5$ almost back to standard PC
        - $\delta < 0.5$ few firms reset price
        - $\delta > 0.5$ closer to flexible prices

Interpreting

- Only difference is just forward-looking expectation of inflation ($E_t\pi_{t+1}$ not $E_{t-1}\pi_t$) but now RatEx no longer means $E(\pi_t - E_t\pi_{t+1}) = 0$ so y_t can be different than $\bar{y}$ and now have room for policy!
- Show this by iterating forwards. People choose prices in frictional setting hence care about future conditions of demand. This is what π encodes
    - $\pi_t = \phi E_t\pi_{t+1} + \lambda x_t = \phi^2 E_t(\pi_{t+2}) + \phi E_t(x_{t+1}) + \lambda x)t = \phi^k E_t(\pi_{t+k}) + \lambda E_t \sum_{i=0}^{k}\phi^i x_{t+i}$
    - As $k \to \infty$ then $\pi_t \to \sum_{i=0}^{\infty}\phi^i x_{t+i}$
- NKPC does not involve lagged inflation π_{t-1} so only persistence comes from x_t

Evidence IRL

- But… this is inconsistent with degree of autocorrelation in inflation
    - Gali and Gertler (1999) suggest "hybrid" version of NKPC that involves fraction of firms ω setting price according to rule of thumb depending on lagged inflation
        - Essentially firms that don't get fairy act dumb
        - $\pi_t = \omega\pi_{t-1} + (1-\omega)E_t\pi_{t+1} + \lambda x_t$
    - But why do firms behave like this?
- If fairy is iid average number of periods before price change is $\frac{1}{\delta}$
    - Using quarterly data initially assumed $\delta = 0.25$
    - Shordone (2002) fits model using macro data and gets $\delta = 0.33$
    - But micro data suggests much closer to fully flexible pricing $\delta = 0.7$

4.2.3 Monetary Policy

- Use three basic equations:
    - [1] NKPC $\pi_t = \phi E_t\pi_{t+1} + \lambda x_t$ where $\lambda = \frac{(\gamma-1)\delta}{1-\delta}$
    - [2] Aggregate demand: $x_t = m_t - p_t$
    - [3] Monetary policy rule. For simplicity assume random-walk $m_t = m_{t-1} + \epsilon_t$
        - Where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$
        - Thus $E_t m_t = E_t m_{t-1} + E_t\epsilon_t = m_t$

- [1] and [2] give $p_t - p_{t-1} = \phi E_t p_{t+1} - \phi p_t + \lambda(m_t - p_t)$
- Rearranges to $\phi E_t p_{t+1} - (1 + \phi + \lambda)p_t + p_{t-1} = -\lambda m_t$
    - This is a second order difference equation with complex solution (see slides)
    - Substituting [3] expectations get $p_t = \zeta p_{t-1} + (1-\zeta)(1-\zeta\phi)\sum^{\infty}(\zeta\phi)^i\, m_t$
    - Note ζ is coefficient of persistence and sum is taking into account future
- Now manipulate
    - Simplify terms $p_t = \zeta p_{t-1} + (1-\zeta)(1-\zeta\phi)m_t \sum^{\infty}(\zeta\phi)^i = \zeta p_{t-1} + \frac{(1-\zeta)(1-\zeta\phi)}{1-\zeta\phi} m_t = p_t - \zeta p_{t-1} + (1-\zeta)m_t$
    - Subtracting p_{t-1} get $\pi_t = (\zeta - 1)p_{t-1} + (1-\zeta)m_t$
    - "Adding and subtracting" $(1-\zeta)m_{t-1}$ get $\pi_t = (1-\zeta)x_{t-1} + (1-\zeta)\epsilon_t$
    - Using AD get $x_t = m_t - p_t = m_t - \zeta p_{t-1} - (1-\zeta)m_t = \zeta m_t - \zeta p_{t-1}$
    - Thus $x_t = \zeta m_t - \zeta m_{t-1} + \zeta m_{t-1} - \zeta p_{t-1} = \zeta\Delta m_t + \zeta x_{t-1} = \zeta x_{t-1} + \zeta\epsilon_t$
- Now have inflation and output dynamics, seeing that ϵ_t can have effect! Output gap is persistent over time and if $\zeta < 1$ gradually decays
    - $\pi_t = (1-\zeta)x_{t-1} + (1-\zeta)\epsilon_t$
    - $x_t = \zeta x_{t-1} + \zeta\epsilon_t$
- Note it fits predictions:
    - Inflation responds to (lag) output gap so there is positive co-movement between two
    - Output gap and inflation respond positively to unanticipated monetary policy shocks
        - MP exploits NKPC relation to affect both
        - Effects of MP persist over time

OVERVIEW RBC vs. NK

	Supply	Demand
RBC	A – marginal cost cannot play a role as perfect competition	Utility function changing Euler condition
NK	Marginal cost $P_t = \frac{\eta}{\eta-1}\frac{W_t}{A_t}$ so either η or A has effect	" " but generally n/a

New Keynesian:

- Households:
    - Decide consumption and leisure allocations.
    - Key implication: intertemporal decisions include "smoothing" motive.
- Two types of firms exist:
    - Intermediate goods firm operate with imperfect competition.
    - Final goods firms operate under perfect competition.
    - Key implications: P > MC, and prices could be "sticky".
- Central bank follows rule to set M (or i).
- Shocks to: technology; monetary and fiscal policy; weather and natural disasters, (COVID?); political; expectations.
- Typically focus on technology and monetary. Why?

- Propagation mechanisms:
    - Intertemporal consumption/investment decision.
    - Labour decisions.
    - Financial mechanisms – kinked budget constraint
- RBC has no role for money or stabilisation policy (only tech)
- NK models couldn't explain 2007/8 crisis. Hence add financial intermediation block
- NK model can't explain COVID crisis. Hence add SIR block
- Heterogenous Agent New Kenysian models (Kaplan, Moll and Violante, 2018) is new third generation model vs. Representative Agent NK 2nd model in lectures
- COVID: NK Kaplan, Moll, and Violante (2020), Guerrieri et al (2020); RBC Eichenbaum et al (2020)
- See Carlo VMAC paper